Random Field Ising Model in four spatial dimensions and beyond

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Barcelona, September 23, 2015

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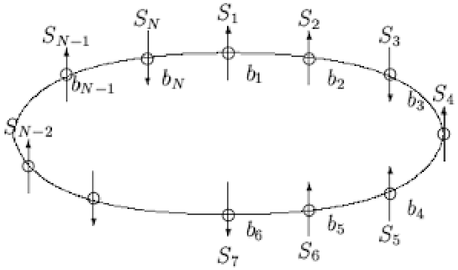
- Basic approach in fundamental Physics: identify minimal model for complex behavior (support: Universality and Wilson's RG).
- The Random-Field Ising Model (RFIM) is a cherised but still not completely understood model for the effects of disorder.

Introduction (II): The standard Ising model

$$H = -J \sum_{\mathbf{x},\mathbf{y}} s_{\mathbf{x}} s_{\mathbf{y}} - h \sum_{\mathbf{x}} s_{\mathbf{x}} \quad , \quad s_{\mathbf{x}} = \pm 1 \, .$$



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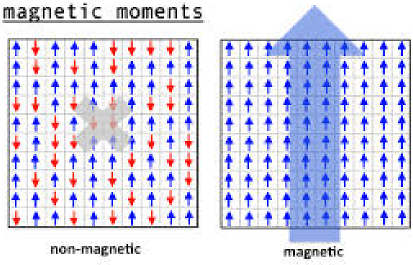

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- Space dimension is an all-important variable.

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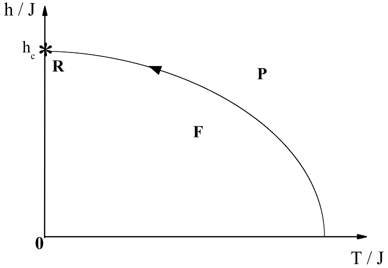

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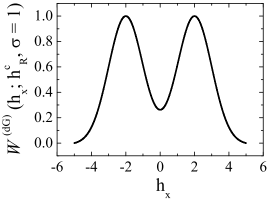

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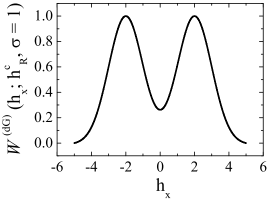

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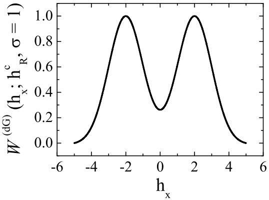

  - $\mathrm{dG}^{(\sigma=2)}$
- Poissonian (P): $\mathcal{W}^{(\mathrm{P})}(h_x; h_R) = \frac{1}{2|h_R|}e^{-|h_x|/h_R}$

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- Perform high statistics in both directions: $L \leq 192$ in $d = 3$ ($L \geq 60$ in $d = 4$), $\#$ disorder samples in the range $(10 - 50) \times 10^6$.

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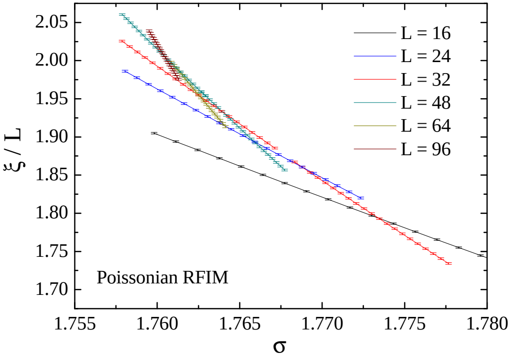
Crossings of the universal ratio ξ/L (from connected Γ_{xy})



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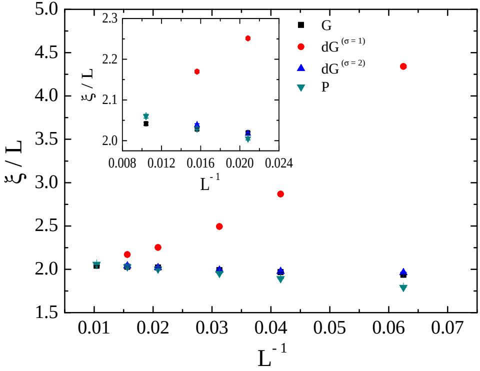
ξ/L at the crossing points: different models differ at fixed L.



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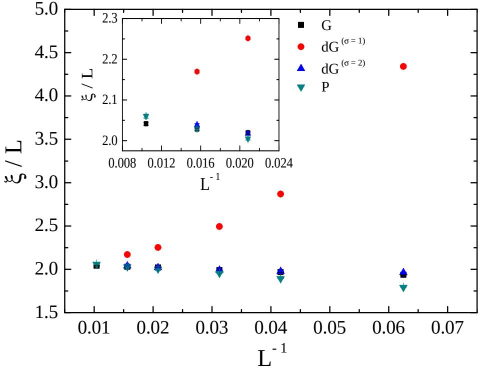
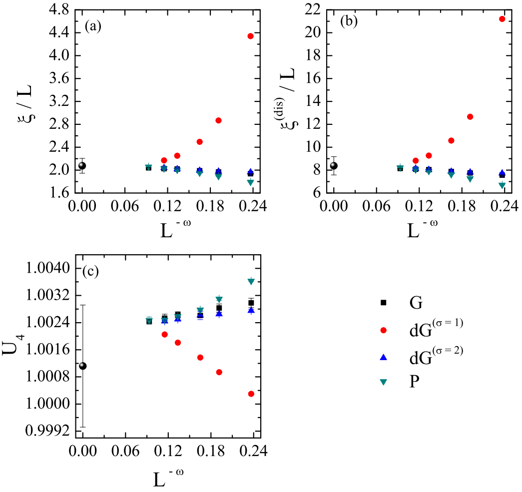


One needs extrapolation to large L.

Universality in the $d = 3$ RFIM

$A + BL^{-\omega} + CL^{-2\omega} + DL^{-3\omega}$; $L_{min} = 24$; $\omega = 0.52 \pm 0.11$;
$\chi^2/dof = 18.83/14$, $Q = 0.17$ (full covariance-matrix!)



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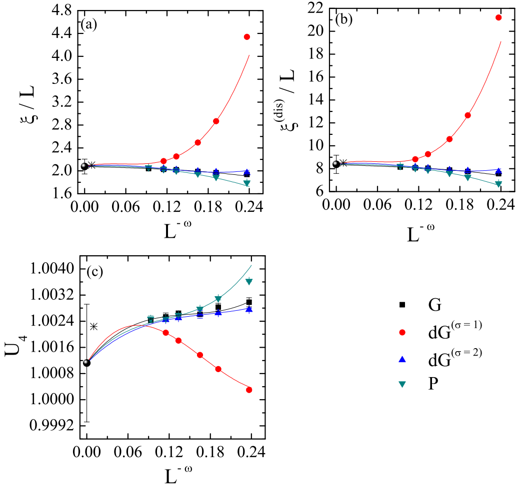
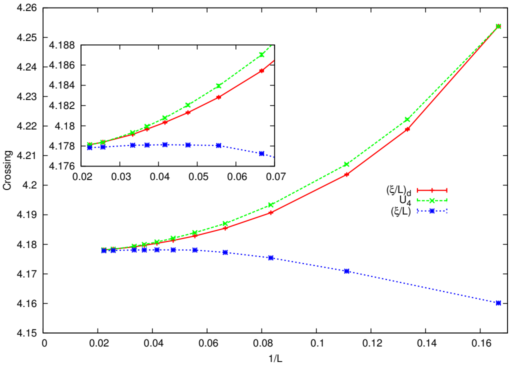
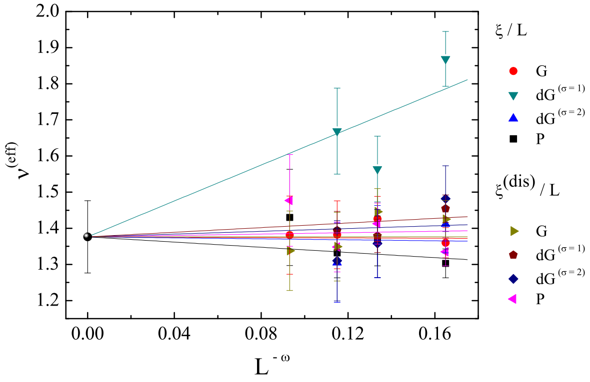


The same approach can be carried out in d=4



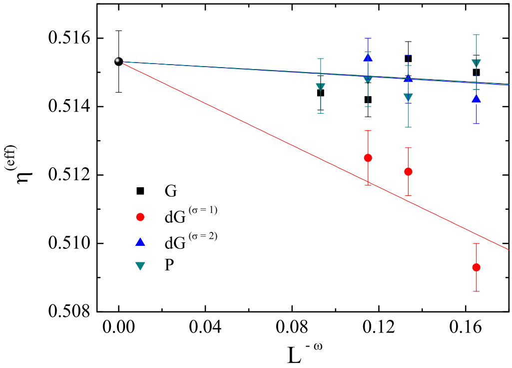
Extrapolation of ν

$\nu_L = \nu + BL^{-\omega}$; $L_{min} = 32$; $\omega = 0.52$;
$\chi^2/dof = 12.52/10$, $Q = 0.25$
Final estimate: $\nu = 1.38 \pm 0.10$



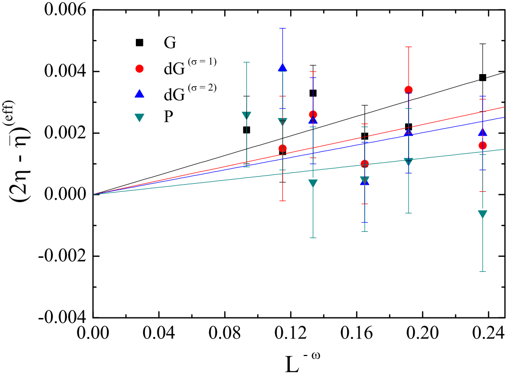
Extrapolation of η

$\eta_L = \eta + BL^{-\omega}$; $L_{min} = 32$; $\omega = 0.52$; $\chi^2/dof = 10/9$, $Q = 0.35$
Final estimate: $\eta = 0.5153 \pm 0.0009$



Extrapolation of $2\eta - \bar{\eta}$

$(2\eta - \bar{\eta})|_L = BL^{-\omega}$; $L_{min} = 16$; $\omega = 0.52$;
$\chi^2/dof = 18.26/18,\ Q = 0.44$



Conclusions

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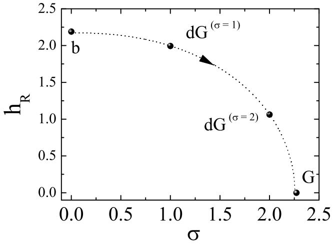

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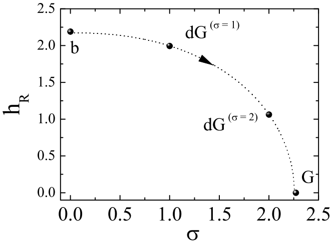

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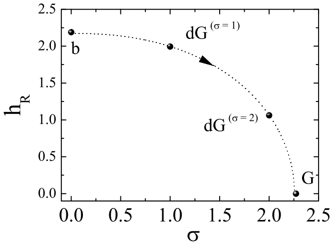

- Existence of **strong scaling corrections** that need to be carefully monitored. Very accurate computation of anomalous dimensions η, $\bar{\eta}$.
- The **two-exponent scaling scenario holds** within an accuracy of two parts in a thousand (2/1000) in $d = 3$. Analysis for $d = 4$ on their way.