

# Using SMT Solving for the Lookup of Infeasible Paths in Binary Programs

University of Toulouse

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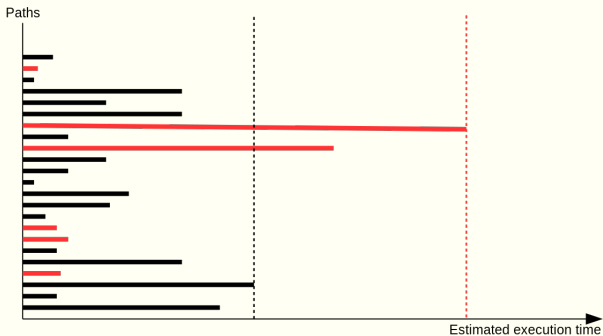
## 4. Conclusion

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# Context

# Improving the WCET estimation

- Find a safe, *tight* timing bound
- Infeasible paths are the main source of overestimation in WCET computation



- Identifying **infeasible paths** refines the WCET estimation

# Working on binary programs

Working directly on binaries is **harder**

- 
- ❖ low expressivity of machine instructions
  - ❖ larger size of program
  - ❖ loosely typed registers
  - ❖ obscure structure of data in memory
  - ❖ ...

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Working directly on binaries is **harder** but is **more adapted**:

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- ❖ ...

- ❖ not mapping properties from source to binaries
- ❖ independent of compiler
- ❖ available source libraries are not required
- ❖ easy injection in WCET computation

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- ❖ short-circuit condition evaluation

```
if (x && a)
    /* ... */
if (x && b)
    /* ... */
```

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Working directly on binaries is **harder** but is **more adapted**:

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- ❖ ...

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```
if (x)
    if (a)
        /* ... */
if (x)
    if (b)
        /* ... */
```



# Analysing the semantics of a binary program

# Semantic instructions

Architecture dependent?

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## Architecture dependent? No!

*ARM machine instructions*

**ADD** r3, r3, #1

*OTAWA semantic instructions*

seti **t1**, 1

add r3, r3, **t1**

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**ADD** r3, r3, #1

**STMDB** sp!, {r4, lr}

*OTAWA semantic instructions*

seti t1, 1

add r3, r3, t1

seti t2, 4

seti t1, 8

sub t1, r13, t1

set t3, t1

store r4, t1, int64

add t1, t1, t2

store r14, t1, int64

add t1, t1, t2

set r13, t3

# Abstract interpretation

- ❖ Maintain an abstract program state for each path
- ❖ Top-to-bottom analysis
- ❖ Inline calls
- ❖ Program states are represented by a *conjunction* of predicates

$$\gamma(\bigwedge \phi_i) = \{x \in S \mid \bigwedge \phi_i(s)\}$$

# Updating the abstract program state

- ❖  $r_{13} = SP_0 + 0$

- ❖  $r_3 = r_1$

(initial state)

$SP_0$  is the initial value of the stack pointer

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- ❖  $t_3 = SP_0 - 8$
- ❖  $[SP_0 - 8] = r_4$

# Updating the abstract program state

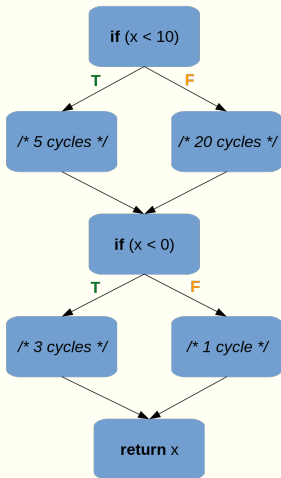
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- ❖  $r_{13} = SP_0 - 8$
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- ❖  ~~$t_2 = 4$~~
- ❖  ~~$t_1 = SP_0 - 4$~~
- ❖  ~~$t_3 = SP_0 - 8$~~
- ❖  $[SP_0 - 8] = r_4$
- ❖  $[SP_0 - 4] = r_{14}$



# Finding infeasible paths

# Example of infeasible path

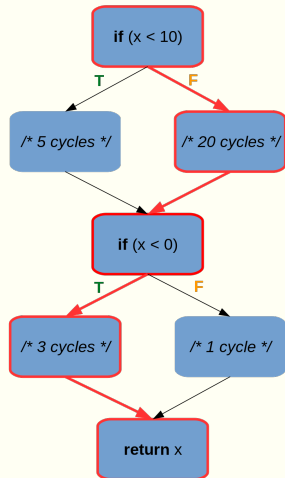


Accounting for all 4 paths,  
WCET = 23 cycles + ...

But:

$$\neg(x < 10) \wedge (x < 0) \models \perp$$

Without the **infeasible path**, WCET = 21 cycles + ...



# Predicates

Example of a simple abstract program state:

❖  $r_0 > 8$

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This program state is **unsatisfiable!**  
(“UNSAT”)

⇒ The current path is infeasible

❖ Example:

$0 \rightarrow 1 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 8 \rightarrow 9$



# Labelled predicates

Label predicates by the basic block(s) that generated them:

❖  $r_0 > 8^{(1,5)}$

❖  $[0x8008] = 0^{(3)}$

❖  $r_{13} = SP_0 - 24^{(4)}$

❖  $r_0 = [SP_0 - 16]^{(9)}$

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❖ Full infeasible path:

$0 \rightarrow 1 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 8 \rightarrow 9$

❖ Minimized infeasible path:

$1 \rightarrow 5 \rightarrow 9$

# SMT solving

**Satisfiability Modulo Theories** solver:

❖ a SAT solver enhanced with multiple theories:

- ❖ Rational/Integer/Booleans
- ❖ Arrays
- ❖ BitVectors
- ❖ ...

⇒ We use Quantifier-Free Linear Integer Arithmetic

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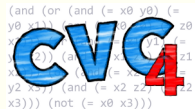
⇒ We use Quantifier-Free Linear Integer Arithmetic

- ❖ receives a list of assertions then seeks a model (satisfiability check)

# UNSAT cores

Some SMT solvers feature UNSAT cores:

- ❖ Triggered when a system is proven unsatisfiable
- ❖ Gives a minimal set of assertions that preserves unsatisfiability



# UNSAT cores

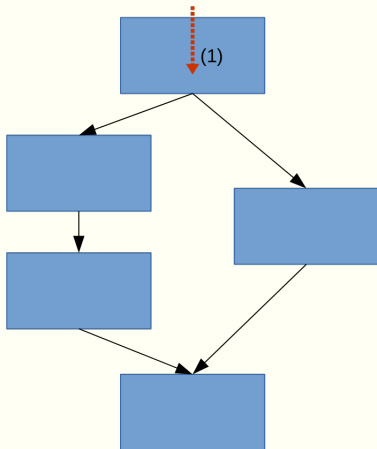
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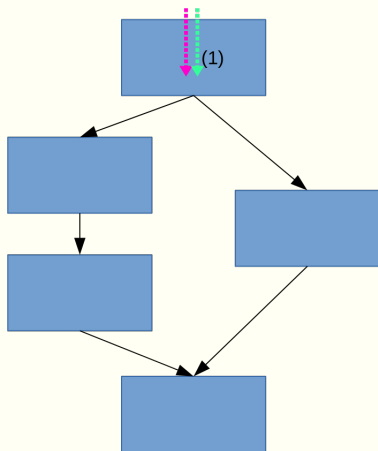
# Macro analysis

- Working List algorithm: *“only process a Basic Block if all paths leading to it have been processed”*



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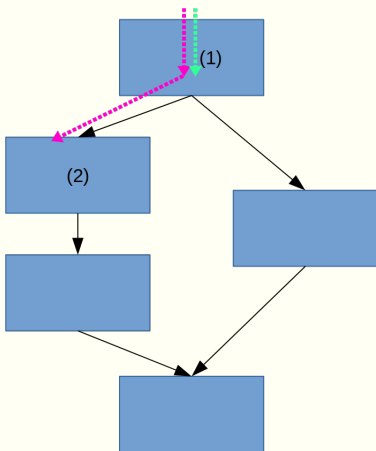
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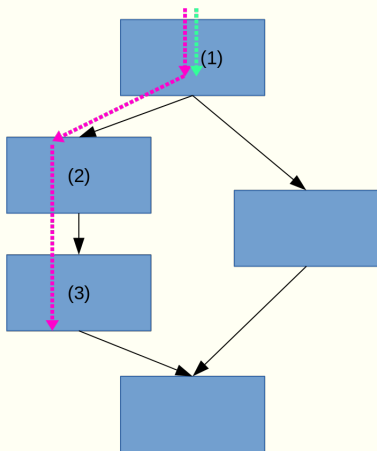
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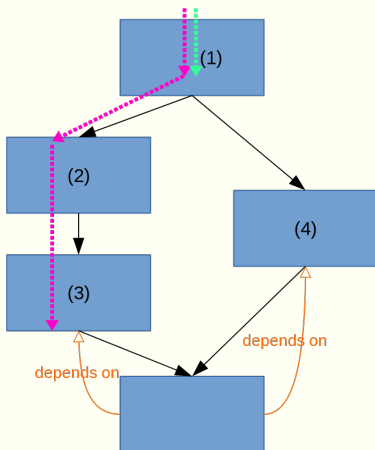
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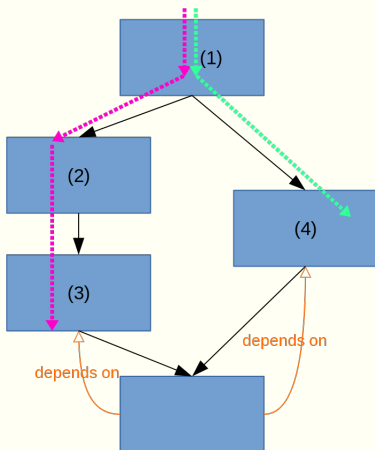
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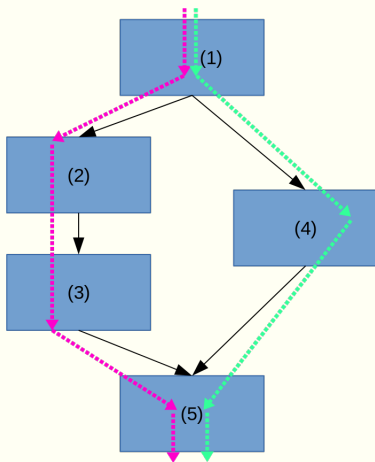
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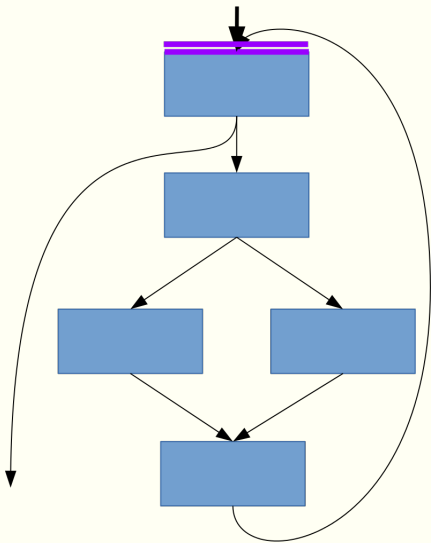


# Macro analysis

- ❖ Working List algorithm: *“only process a Basic Block if all paths leading to it have been processed”*
- ❖ Loops:
  - ❖ Iterate and merge with previous state until fixpoint is reached
  - ❖ When a fixpoint is reached, enable SMT checks to find infeasible paths valid at every iteration

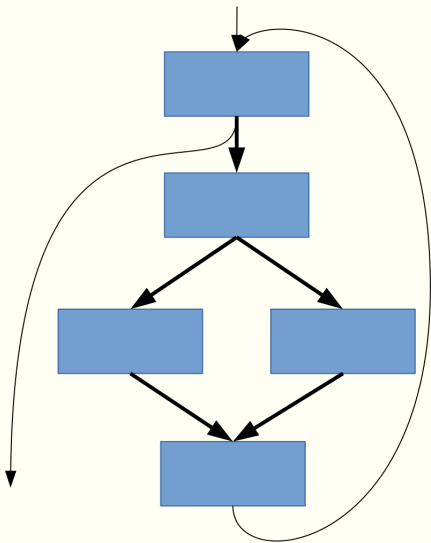
# Processing loops

- ❏ To start things off, merge all incoming states



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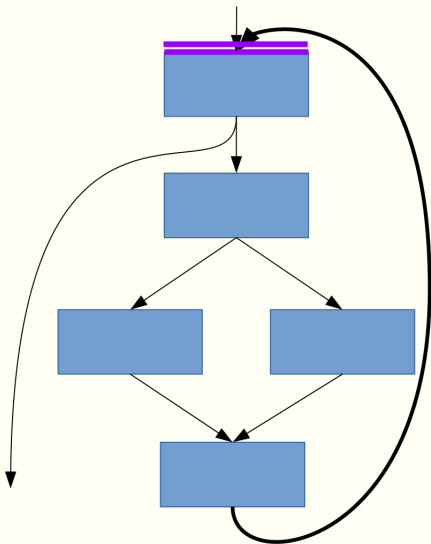
- ❏ To start things off, merge all incoming states
- ❏ Parse the loop body normally





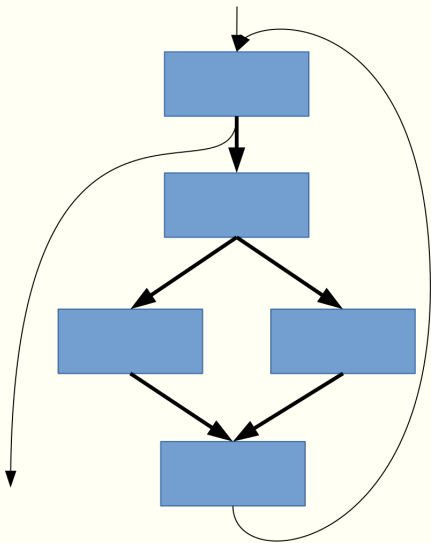
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- ❖ Then merge with the previous state



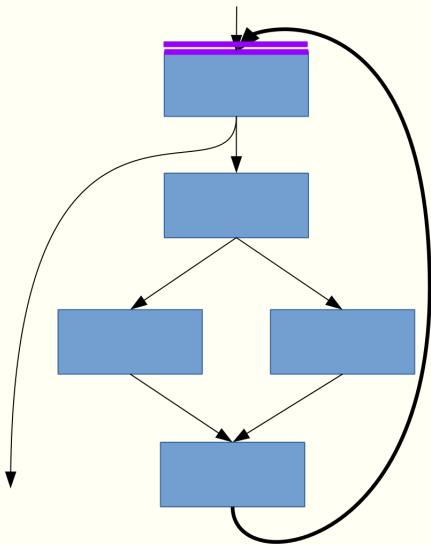
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- ❖ To start things off, merge all incoming states
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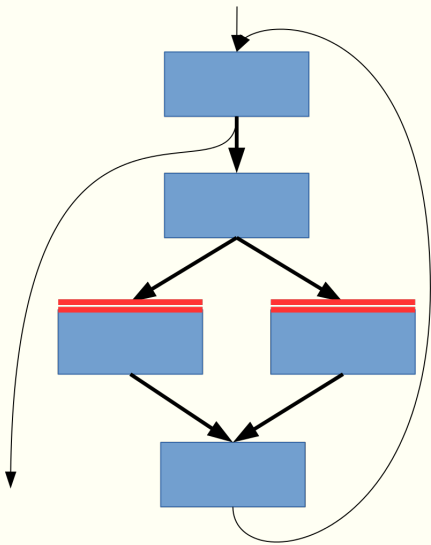
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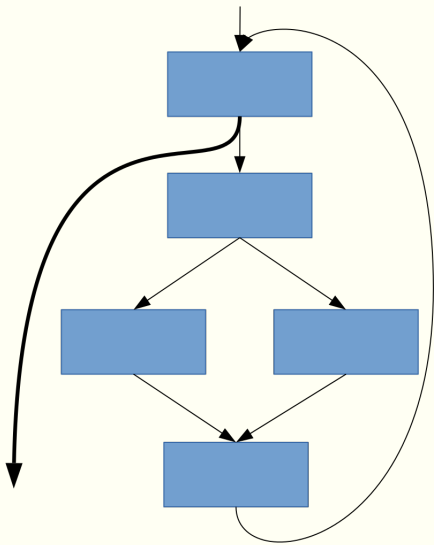
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- ❖ Do SMT checks



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- ❖ Repeat until fixpoint is reached
- ❖ Do SMT checks
- ❖ Exit the loop



# Merging

A very rough merging algorithm: predicate set intersection

$$\text{❖ } r_{13} = SP - 4$$

$$\text{❖ } r_0 = [SP - 8]$$

$$\text{❖ } r_1 = 0$$

$$\text{❖ } r_2 = 0$$

$$\text{❖ } [0x8008] = 16$$

$$\text{❖ } r_{13} = SP - 4$$

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$$\text{❖ } r_1 = 1$$

$$\text{❖ } r_2 > 0$$

becomes:

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becomes:

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# Mälardalen benchmarks

Benchmark	BB (#)	Time (s)	IPs found with minimization	without minimization
SMALL BENCHMARKS (NO MERGING REQUIRED)				
ndes	57	0.267	0	0
expint	70	0.748	14	34
edn	75	0.537	2	2
prime	118	4.368	22	43
compress	122	1.801	10	19
select	136	45.598	4	8
qsortexam	155	28.201	9	12
adpcm	323	0.074	3	3
LARGE BENCHMARKS (MERGING REQUIRED)				
ud	153	17.477	6	23
minver	449	188.339	4	16
statemate	453	193.849	16	22
ludcmp	632	143.088	11	510
nsichneu	754	250.385	4586	8620
qurt	2777	773.805	3	3
lms	3098	915.434	259	2376
fft1	6123	2223.125	25	815



# Conclusion

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- Brutal state merging
- Time calculation explosion

# Future works

- ▣ Program slicing

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- Experimentations to estimate the impact on the WCET

# Questions?

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			1 edge	Minimized	Non-minimized	Non-minimized
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