3D ELASTIC FULL WAVEFORM INVERSION:
TOWARD REFLECTION BASED INVERSION

Jean Kormann, BSC-CNS CASE dpt.
1. Introduction to geophysical exploration
2. Full Waveform Inversion
3. Application to real dataset
INTRODUCTION TO GEOPHYSICAL EXPLORATION
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Top500.org – November 2015

6.7  6.0  5.0  3.0  2.2
Oil and Gas Exploration
Oil and Gas Exploration

- Land: 1-15 M$
- Shallow water: 30 M$
- Deep water: 100 M$

Barcelona Supercomputing Center
Centro Nacional de Supercomputación
Oil and Gas Exploration

SUPERCOMPUTING

Processing

Interpretation

- Land: 1-15 M$
- Shallow water: 30 M$
- Deep water: 100 M$
Transforming data into images
Transforming data into images
Transforming data into images

Range (km)

shotgather

Time-domain image

Range (km)

Conventional
Inverse problem

Model $K$ -> Constitutive Equation -> Synthetic Data -> Misfit Function

- Global maximum
- Local maximum
- Local minimum
- Global minimum
Inverse problem

Model $K$ → Constitutive Equation

Traces (f WITHOUT (left))

Iteration 1

Iteration 10

Iteration 20

Final Velocity (f=15Hz - L1 - NCG) WITHOUT (left) and WITH MULTISCALE

Iteration 1

Iteration 10

Iteration 20
Data

Trace
Initial guess
Inversion

![Graph with x 100,000,000]
Adjoint Method

Elastic wave equation:

\[
\frac{\partial^2 u_i}{\partial t^2} - \frac{\partial}{\partial x_j} \left( c_{ijkl} \frac{\partial u_k}{\partial x_l} \right) = f_i
\]

\[u(x, t=0) = 0\]

\[\frac{\partial u(x, t=0)}{\partial t} = 0\]

Lagrangian equation:

\[
\Lambda = \frac{1}{2} \sum \int_0^T \| u(x_r, t) - d(x_r, t) \|^2 dt - \int_0^T \int_\Omega \lambda_i \left[ \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial}{\partial x_j} \left( c_{ijkl} \frac{\partial u_k}{\partial x_l} \right) - f_i \right] dV dt
\]
Derivative of $\Lambda$ respect to parameter and field $u$:

$$\partial \Lambda = - \int_0^T \int_{\Omega} \delta \rho \left[ \lambda_i \frac{\partial u_i}{\partial t} \right] dV - \int_0^T \int_{\Omega} \delta \rho \frac{\partial \lambda_i}{\partial t} \frac{\partial u_i}{\partial t} dV dt$$ \hspace{1cm} (1)

$$\partial \Lambda = \int_0^T \int_{\Omega} \left[ u_i(x_r, t) - d_i(x_r, t) \right] \delta u_i dV dt + \int_0^T \int_{\Omega} \frac{\partial u_i}{\partial t} \left( \frac{\partial}{\partial x_j} \left( c_{ijkl} \frac{\partial \lambda_k}{\partial x_l} \right) \right) dV dt - \int_0^T \int_{\Omega} \rho \lambda_i \frac{\partial^2 u_i}{\partial t^2} dV dt \hspace{1cm} (2)$$

$$\partial \Lambda = \int_0^T \int_{\Omega} \left[ \lambda_i \frac{\partial}{\partial t} \frac{\partial \delta u_i}{\partial t} \right] dV + \int_0^T \int_{\partial \Omega} \lambda_i \frac{\partial}{\partial x_j} \left( c_{ijkl} \frac{\partial \lambda_k}{\partial x_l} \right) n_j dS \hspace{1cm} (3)$$
Setting time and boundaries condition to $\lambda$ leads to:

$\lambda(x, T) = 0$

$\frac{\partial \lambda(x, T)}{\partial t} = 0$

$\lambda_j(x,t)n_j = 0, \, x \in \partial \Omega$

Finally we get the following expression:

$$\delta \Lambda = \int_0^T \int_\Omega (u_i(x_r,t) - d_i(x_r,t)) \delta u_i dV dt - \int_0^T \int_\Omega \delta u_i \left[ \frac{\partial^2 \lambda_i}{\partial t^2} - \frac{\partial}{\partial x_j} \left( c_{ijkl} \frac{\partial \lambda_k}{\partial x_l} \right) \right] dV dt$$

$$+ \int_0^T \int_\Omega \delta c_{ijkl} \frac{\partial \lambda_i}{\partial x_j} \frac{\partial u_k}{\partial x_l} dV dt - \int_0^T \int_\Omega \delta \rho \frac{\partial \lambda_i}{\partial t} \frac{\partial u_i}{\partial t} dV dt$$
FWI workflow

Starting Model 3 → Prop. → Synthetic Data Set → Filter → Observable Data Set

L2 Norm $\int_0^T \varepsilon(t) \, dt$

Misfit Function $\varepsilon(t) = 1/2(d_{\text{obs}}(x,t) - d_{\text{Synt}}(x,t))^2$

Adjoint Source

$-\frac{\partial}{\partial x} \varepsilon = -[d_{\text{obs}}(x,t) - d_{\text{Synt}}(x,t)]$

Adjoint Problem

Model Gradient $\nabla \mathcal{I}$

Optimization Method

New Model 3

Final Model 3

Prop.

Optimization Part

Adjoint Method Part
Resuming...

Loop Control

Frequency loop, filtering

Gradient iteration

Adjoint Method: Gradient computation

Windowing

Receivers selection

Line Search: Perturbed model
FULL WAVEFORM INVERSION
SEG/EAGE Overthrust Model

3249 sources, 3249 receivers
Staggered geometry
Receivers and sources are at depth 50 m
8 s of simulation (250 GB of data)
Ricker with central frequency 10 Hz
Constant density
$\Delta d$ is 20 m (3.2 x 16 x 16 km$^3$) and $\Delta t$ is 0.0013 s.
8th order FDTD in space and 2nd in time
Multi-Scale strategy: 4 low-pass filters to linearize problem
Mesh adapted to frequency
Multi-shooting for data reduction: 3249 shots → 56 supershots
Novel in-house preconditioner
Non-linear Conjugate Gradient method
No fixed part of the model
Neither sources nor receivers are coincident with the mesh grid
Noise free dataset
8 hours and 57 nodes for 80 iterations
BSIT FWI: multi-scale/multi-grid

START

1 Hz

2 Hz

4 Hz

8 Hz

TARGET
BSIT FWI: inverted depth slices

Z=345 m

Z=1600 m

Z=2150 m
BSIT FWI: inverted velocity profiles

X-Line

Y-Line
• **Modelling:**
• 6 s simulation with 10 Hz source frequency peak, 5041 shots, 3x16x16 Km3 (Vp and Vs models), fixed density
• SSG grid with mimetic operator for free-surface modelization
• ~48000 cpus during 3 days (~ 3000 nodes)
• Run on BSC-CNS *Mare Nostrum III* supercomputer
BSIT FWI: 3D Multi-Parameters Inversion
BSIT FWI: 3D Multi-Parameters Inversion

• **Inversion:**
  - Simultaneous inversion of Lame's parameters, fixed density
  - 4 frequencies: 2.6, 3.4, 5.2, and 6.8 Hz; 10 Iterations/freq
  - Efficient Multi-Scale, Multi-grid implementation for mesh size reduction
  - Free-surface included, no phase selection, no fixed parts of the models
  - 3296 shots used for inversion
  - *Dynamic Offset Control* for domain reduction
  - 72 hours using 1408 cpus (88 nodes) on BSC-CNS *Mare Nostrum III* supercomputer for 40 iterations
BSIT FWI: 3D Multi-Parameters Inversion
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BSIT FWI: 3D Multi-Parameters Inversion
DOES IT REALLY WORK?
BSIT FWI: Real Dataset Application

**DATASET:**
- 52 shots of 2 s (red dot)
- 48 receivers $a_z$ component only (white dot)
- Vibroseis source

**INVERSION**
- 0.4 s
- Topography
- Source extract from dataset
- $V_p$ starting model obtained from TTT
BSIT FWI: Real Dataset Application

**DATASET:**
- 52 shots of 2 s (red dot)
- 48 receivers a \(_{z}\) component only (white dot)
- Vibroseis source
- Utile bandwidth: 10-40 Hz

**INVERSION**
- 0.4 s
- Topography
- Source extract from dataset
- \(V_p\) starting model obtained from TTT
- 2 frequencies: 15 and 20 Hz
- 27 nodes, 24 hours; 2 nodes per shots
BSIT FWI: Real Dataset Application

Conventional Grid Cell

SSG Cell

FSG Cell

Acoustic

Elastic Triclinic

Elastic
ISO/VTI/HTI/...
BSIT FWI: Real Dataset Application
BSIT FWI: Real Dataset Application
BSIT FWI: Real Dataset Application

Below are graphs showing seismic wave velocities ($V_s$ and $V_p$) as a function of depth. The graphs illustrate the comparison between different models, such as SCV6, Inverted, and Starting. Additional traces from different shots (Shot 20314) are also displayed, showing normalized traces for each case.
BSIT FWI: Real Dataset Application
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