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Hybrid Finite Difference-pseudospectral Method for 3D RTM in TTI Media

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SUMMARY

We propose a new anisotropic wave equations system for 3D TTI media, which is an extension to 3D VTI media and 2D TTI media equations of Zhou et al. This system is based on two 3D rotations which permit us to deduce 3D TTI equations from 3D VTI equations. We propose to use a hybrid Finite Difference (FD) pseudospectral algorithm to solve it, this mainly consists of forward-backward 2D FFT in lateral dimensions (x-y plane) and 1D FD in the depth dimension. This algorithm allows us to get high order accuracy and simplifies the computation of the cross derivatives of the TTI equations. In this work, we develop the 3D TTI equation formalization, we also describe the implementation of the method to solve the proposed 3D TTI equations. To validate our proposal, we carry out impulse response experiments for modeling and migration.

Introduction

Conventional isotropic methods for seismic data processing are subject to errors which lead to lower resolution and misplaced images of sub-surface structures. For production-delineation of reservoirs, it is clear that higher-resolution methods must take anisotropy into account [5,6]. In this work, we consider two assumption cases of transversely isotropic (TI) media: VTI and TTI media. VTI yields for TI media with vertical axis of symmetry (observed in sedimentary basins). TTI assumption means a TI media with a tilted axis of symmetry (observed in regions with anticlinal structures and/or thrust sheets). Currently, Zhou et al. [7] proposed an equivalent coupled system to Alkhalifah's [1] "acoustic" approximation for 3D VTI media. This system of lower-order adds even more simplicity to the Alkhalifah´s approximation already attractive for modeling and migration. In [8], Zhou et al extended their idea to the 2D TTI case. In this work, we combine Zhou and al ideas for 3D VTI and 2D TTI to deduce a new 3D TTI equations system applying two 3D rotations to the 3D VTI equations.

To solve this system, we use partially the attractive Pseudospectral (PS) method proposed by Kosloff [4], which introduces an alternative to the two classic Finite Difference (FD) and Finite Element (FE) methods. The advantage of this method is that the spatial discretization can be coarser and more accurate [2] in comparison with FE and FD methods. For these reasons, the PS method is attractive for modeling of seismic wave propagation in a heterogeneous medium [3]. Moreover, choosing PS instead of FD, simplifies the computation of cross derivatives present in the proposed 3D TTI system. However because efficient 3D FFT is not yet available, we propose to use a hybrid FD-PS algorithm to solve it, this mainly consists of forward-backward 2D FT in lateral dimensions (x-y plane) and 1D FD in the depth dimension.

Anisotropic Acoustic Wave Equation in 3D TTI Media

Zhou et al. [7] proposed a 3D anisotropic acoustic equation for VTI media and a 2D anisotropic acoustic equation for TTI media [8]. Their governing systems of equations are based on the same dispersion relation of Alkhalifah [1], but by introducing an auxiliary function, the original fourth-order differential equation becomes a coupled system of lower-order differential equations, which are similar to isotropic equations and therefore is easier for implementation than Alkhalifah's. Their 2D TTI acoustic equation can be expressed as:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - (1+2\delta)Hp - H_0\frac{\partial^2 p}{\partial z^2} = (1+2\delta)Hq, \qquad (1)$$

$$\frac{1}{c^2}\frac{\partial^2 q}{\partial t^2} - 2(\varepsilon-\delta)Hq = 2(\varepsilon-\delta)Hp \quad . \qquad (2)$$

where the differential operator H and H_0 are defined as:

$$H = \cos^2(\theta)\frac{\partial^2}{\partial x^2} + \sin^2(\theta)\frac{\partial^2}{\partial z^2} - \sin(2\theta)\frac{\partial^2}{\partial x\partial z} \qquad (3)$$

$$H_0 = \sin^2(\theta)\frac{\partial^2}{\partial x^2} + \cos^2(\theta)\frac{\partial^2}{\partial z^2} + \sin(2\theta)\frac{\partial^2}{\partial x\partial z}, \qquad (4)$$

where θ is the angle of the symmetry axis with respect to the z axis. Unlike the VTI equations as shown in Zhou et al.[7], cross-derivative terms appear in equations (3) and (4) because of the TTI characteristics. Equation (1) can be considered as a hyperbolic wave equation for elliptical anisotropy but with correction term to compensate for the anisotropy loss of the TTI media . Equation 2 can be considered as the additional expansion or contraction of the wavefront in the lateral directions.

In the following, we will extend the 2D TTI case into 3D TTI by axis rotations of the original VTI equations. We can derivate a 3D anisotropic acoustic wave equation for TTI media by combining two rotations. The first one is a rotation of angle θ along y-axis, and the second rotation along angle φ in the x-axis. We apply this variable change to the 3D anisotropic acoustic equation for VTI media. The 3D anisotropic acoustic equation for TTI media can be formulated as in 2D [equations (1) and (2)] but with the following expressions for the two differential operators H and H_0.

$$H = A\frac{\partial^2}{\partial x^2} + (B + AD)\frac{\partial^2}{\partial y^2} + (A - DB)\frac{\partial^2}{\partial z^2} - CH\frac{\partial^2}{\partial x \partial y} - CG\frac{\partial^2}{\partial x \partial z} - AF\frac{\partial^2}{\partial y \partial z} \quad (5)$$

$$H_0 = B\frac{\partial^2}{\partial x^2} + AE\frac{\partial^2}{\partial y^2} + AD\frac{\partial^2}{\partial z^2} + CH\frac{\partial^2}{\partial x \partial y} + CG\frac{\partial^2}{\partial x \partial z} + AF\frac{\partial^2}{\partial y \partial z} \quad (6)$$

with

$$A = \cos^2(\theta) \quad B = \sin^2(\theta) \quad C = \sin(2\theta), \quad D = \cos^2(\varphi), \quad E = \sin^2(\varphi)$$

$$F = \sin(2\varphi), \quad G = \cos(\varphi), \quad H = \sin(\varphi)$$

In equations (5) and (6), if we set θ then φ to zero, we can check that the formulation of the differential operators is reduced almost like for the 2D TTI case [equations (3) and (4)]. To treat the difference, there is an additional second derivative which corresponds to the third direction not tilted (y in the case φ=0) and (x in the case θ=0). Thus, in numerical experiments, we should observe results similar to 2D TTI ones for those particular values of θ and φ. Note that in general, θ and φ are function of the space but assuming they vary slowly their derivatives can be neglected in the equations.

Time and spatial discretization

The second-order FD stencil of equations (1) and (2) in time direction can be symbolically written as:

$$p^{n+1} = 2p^n - p^{n-1} + c^2 \Delta t^2 \left[(1+2\delta)H(p^n + q^n) + H_0 p^n\right] \quad (7)$$

$$q^{n+1} = 2q^n - q^{n-1} + c^2 \Delta t^2 \left[2(\varepsilon - \delta)H(p^n + q^n)\right] \quad (8)$$

To reduce numerical dispersions, the space derivatives in equations should be approximated by higher order schemes. Obviously, because of the cross-derivative term, FD is found to be too complex for implementation. And because of the 3D FFT is hard to be implemented efficiently, we decided to use hybrid-pseudospectral algorithms. i.e., we choose to apply pseudo-spectral algorithm in lateral directions (x and y) and FD scheme in depth direction (z). In this way, we remove the need to compute the cross derivatives.

The hybrid formulation relies on the following algorithm:

- First we compute the partial forward FT in x and y of p^n and q^n:

$$p^n(x,y,z,t) \to \hat{p}^n(k_x,k_y,z,t), \; q^n(x,y,z,t) \to \hat{q}^n(k_x,k_y,z,t)$$

- Second we calculate the derivatives in x and y needed in H and H_0 in the wave number domain. We define

$$\hat{H}_1 = (-Ak_x^2 - (B+AD)k_y^2 + (CH)k_x k_y)(\hat{p}^n + \hat{q}^n), \; \hat{H}_2 = -(CGik_x + AFik_y)(\hat{p}^n + \hat{q}^n)$$

$$\hat{H}_{01} = -(Bk_x^2 + AEk_y^2 + (CH)k_x k_y)\hat{p}^n, \; \hat{H}_{02} = (CGik_x + AFik_y)\hat{p}^n$$

- Third we calculate the backward FT of $\hat{H}_1, \hat{H}_2, \hat{H}_{01}$ and $\hat{H}_{02}$ .Then equations (7,8) give

$$p^{n+1} = 2p^n - p^{n-1} + c^2\Delta t^2\left[(1+2\delta)(H_1 + \frac{\partial H_2}{\partial z} + (A - BD)\frac{\partial(p^n + q^n)}{\partial z^2}) + H_{01} + \frac{\partial H_{02}}{\partial z} + AD\frac{\partial^2 p^n}{\partial z^2}\right] \quad (9)$$

$$q^{n+1} = 2q^n - q^{n-1} + c^2\Delta t^2\left[2(\varepsilon - \delta)(H_1 + \frac{\partial H_2}{\partial z} + (A - BD)\frac{\partial(p^n + q^n)}{\partial z^2})\right] \quad (10)$$

- To finish, we calculate the derivatives in z with FD schemes.

Numerical Results

To validate our hybrid pseudospectral-FD method for 3D anisotropic acoustic wave equation in TTI media, we run impulse response experiments for an homogeneous isotropic media and an homogeneous TTI media. The given TTI media has the anisotropic parameters defined by c=2000m/s, ε=0.2 and δ=0.05, but with various rotation angles θ and φ of the symmetry axis. The source wavelet is Ricker wavelet with a peak of frequency as 30 Hz. Figure 1 (page 4) shows the snapshots generated by our 3D TTI wave equation for the isotropic case ($\varepsilon=\delta=0;\theta=\varphi=0$) and for the anisotropic case ($\varepsilon=0.2,\delta=0.05$) with the following combinations of the two angles ($\theta=\varphi=0$ (VTI case), $\theta=45;\varphi=0$, $\theta=90;\varphi=0$, $\theta=\varphi=45$ degrees). Figure 2 show the results of impulse responses of pre-stack migration for the same angle combinations as presented for the modeling. The location for the source and receiver are 500m apart. Figures 1 and 2 show 2D cuts in the middle of y dimension of the domain. We have checked that the impulse responses are the same in the mentioned 2D cut with $\theta = \alpha, \varphi = 0$, and in the middle of x dimension with $\theta = 0, \varphi = \alpha$. These results are consistent with the ones obtained by Zhou *et al.* in 2D TTI. The artifacts in the center of the image, for some particular cases, exist due to the slower-traveling waves inherent in the solution.

Conclusion

We have described a new anisotropic wave equation for 3D TTI media, which is an extension to 3D VTI media and 2D TTI media equations of Zhou *et al.* The newly developed anisotropic wave equation provides a good kinematics approximation to the elastic equation. This new wave equation uses lower-order derivatives than Alkhalifah's, and it is convenient not only for computationally efficiency but also for modeling and migration algorithm. To solve this equation, we proposed an hybrid FD-pseudospectral method. The use of pseudospectral method is appealing because it corresponds to an high order accuracy scheme, and at the same time allows us to use coarse meshes, also simplifies the computation of the cross derivatives. Impulse response experiments for modeling and pre-stack migration show the validity of the algorithm. Parallelization and use of the algorithm for high performance computation is underway.

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Fig 1. Impulse response for modeling. From top to bottom, they correspond to isotropic media, 3D TTI media with angle combinations θ=φ=0 (VTI), θ=45;φ=0, θ=90;φ=0, θ=φ=45 degrees

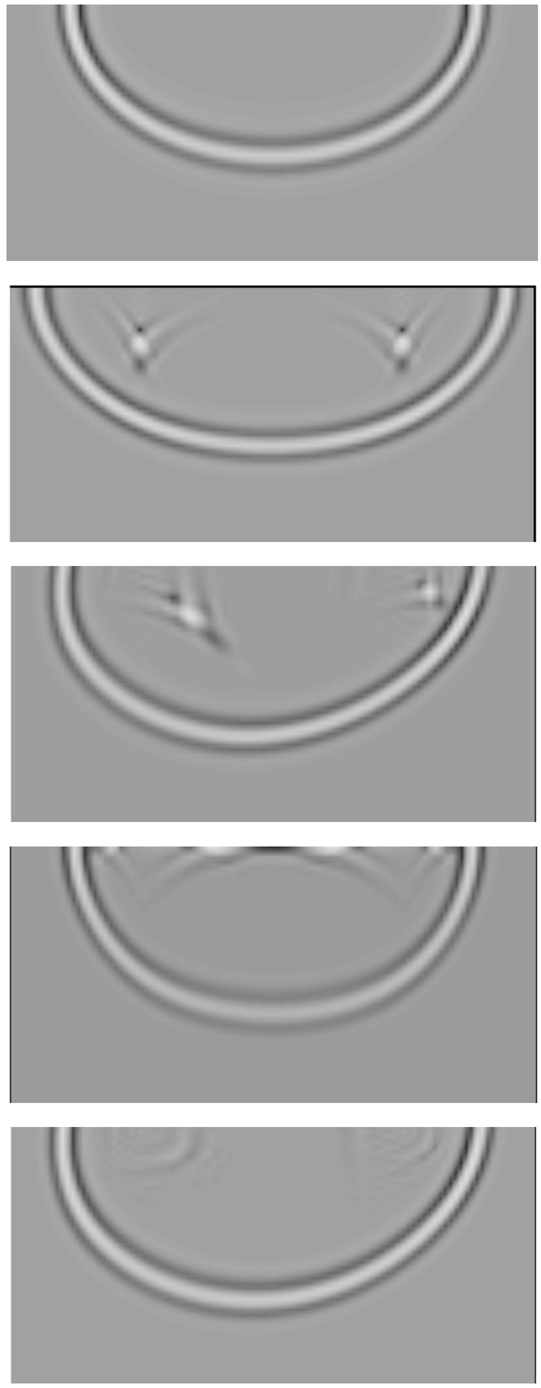

Fig 2. Impulse responses of pre-stack migration. From top to bottom, they correspond to isotropic media, 3D TTI media with angle combinations θ=φ=0 (VTI), θ=45;φ=0, θ=90;φ=0, θ=φ=45 degrees