3D reverse-time migration with Hybrid Finite Difference-pseudospectral Method

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SUMMARY

We propose a 3D reverse-time migration (RTM) using an hybrid Finite Difference (FD) pseudospectral algorithm to solve the two-way acoustic equation. This mainly consists of forward-backward 2D FFT in lateral dimensions (x-y plane) and 1D FD in the depth dimension. This algorithm allows us to get high order accuracy and simplifies the computation of cross derivatives. Therefore our RTM allows to deal with the case of 3D isotropic media, VTI media (Zhou et al., 2006b) and 3D TTI media. The 3D TTI media case lies on a new anisotropic wave equations system (Lesage et al., 2008), which is an extension of 3D VTI media (Zhou et al., 2006b) and 2D TTI media equations (Zhou et al., 2006a) of Zhou *et al.* . This system is based on the combination of two 3D rotations which permits us to deduce 3D TTI equations from 3D VTI equations. In this work, we recall the formalization for 3D isotropic, 3D VTI and 3D TTI media, we also describe the implementation of the method to solve the proposed 3D TTI equations. To validate our proposal, we carry out impulse response experiments for modeling and migration.

INTRODUCTION

Thanks to high performance computing environments, production imaging for geophysic prospection of earth depth below sea bottom can now be done with 3D isotropic RTM. It uses the two-way acoustic isotropic wave equation and is relevant for challenging geological environment because it does not suffer from dip limitation. But seismic anisotropy in dipping shales can result in imaging and positionning problems for underlying structures. For production-delineation of reservoirs, it is clear that higher-resolution methods must take anisotropy into account (Thomsen, 1986) (Tsvanskin, 2001). In this work, we consider three RTM cases : isotropic and two transversely isotropic cases (VTI and TTI media). VTI yields for TI media with vertical axis of symmetry (observed in sedimentary basins). TTI assumption means a TI media with a tilted axis of symmetry (observed in regions with anticlinal structures and/or thrust sheets). Currently, Zhou *et al.* (Zhou et al., 2006b) proposed an equivalent coupled system to Alkhalifah's (Alkhalifah, 2000) "acoustic" approximation for 3D VTI media. This system of lower-order adds even more simplicity to the Alkhalifah's approximation already attractive for modeling and migration. In (Zhou et al., 2006a), Zhou *et al.* extended their idea to the 2D TTI case. In this paper, we recall the combination of Zhou et al ideas for 3D VTI and 2D TTI which produces a new 3D TTI equations system (Lesage et al., 2008). This system is based on the combination of two 3D rotations which permits to pass from the vertical axis to the tilted one.

To solve the different acoustic equations, we use partially the attractive Pseudospectral (PS) method proposed by Kosloff (Kosloff and Baysal, 1982), which introduces an alternative to the two classic Finite Difference (FD) and Finite Element (FE) methods. The basic idea of the pseudospectral is to expand the field quantities by the mean of a discrete Fast Fourier Transform (FFT) and to compute analytically the spatial derivation in the wave number domain. The advantage of this method is that the spatial discretization can be coarser and more accurate (Fornberg, 1987) in comparison with FE and FD methods. For these reasons, the PS method is attractive for modeling of seismic wave propagation in a heterogeneous medium (Furumura et al., 1998). Moreover, choosing PS instead of FD, simplifies the computation of cross derivatives present in the case of the 3D TTI system. However because efficient 3D FFT is not yet available, we propose to use a hybrid FD-PS algorithm to solve it, this mainly consists of forward-backward 2D FT in lateral dimensions (x-y plane) and 1D FD in the depth dimension.

ISOTROPIC AND TRANSVERSELY ISOTROPIC RTM

Reverse-time migration uses the well established imaging condition $I(z,x,y)$ of zero-lag cross correlation between forward propagated source wavefield $S(z,x,y,t)$ and the backward propagated receiver wavefield $R(z,x,y,t)$ summed over the sources s (Biondi and Shan, 2002) where z, x and y denote depth, horizontal and lateral axis, respectively and t.

Isotropic Acoustic Wave Equation

For the isotropic case, the two wavefields are propagated with the following 3D isotropic wave equation for the pressure wave p:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \Delta p, \tag{1}$$

with c the wave velocity.

Anisotropic Acoustic Wave Equation in 3D VTI Media

For the 3D VTI case, we use the 3D anisotropic acoustic equation proposed by Zhou *et al.* (Zhou et al., 2006b) for VTI media:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = (1+2\delta)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(p+q)+\frac{\partial^2 p}{\partial z^2}, \tag{2}$$

$$\frac{1}{c^2}\frac{\partial^2 q}{\partial t^2} = 2(\varepsilon-\delta)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(p+q) \tag{3}$$

Their governing systems of equations are based on the same dispersion relation as Alkhalifah (Alkhalifah, 2000), but by introducing an auxiliary function q, the original fourth-order differential equation becomes a coupled system of lower-order differential equations, which is similar to isotropic equation and therefore is easier to implement than Alkhalifah's. Equation (2) can be considered as a hyperbolic wave equation for elliptical anisotropy but with correction term to compensate for the anisotropy loss of the VTI media. Equation (3) can be considered as the additional expansion or contraction of the wavefront in the lateral directions.

Anisotropic Acoustic Wave Equation in 3D TTI Media

Zhou *et al.* (Zhou et al., 2006b) proposed also a 2D anisotropic acoustic equation for TTI media (Zhou et al., 2006a). Their 2D TTI acoustic equations system writes as:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - (1+2\delta)Hp - H_0\, p = (1+2\delta)Hq, \quad (4)$$

$$\frac{1}{c^2}\frac{\partial^2 q}{\partial t^2} - 2(\varepsilon-\delta)Hq = 2(\varepsilon-\delta)Hp. \quad (5)$$

where the differential operator H and H_0 are defined as:

$$H = cos^2(\theta)\frac{\partial^2}{\partial x^2} + sin^2(\theta)\frac{\partial^2}{\partial z^2} - sin(2\theta)\frac{\partial^2}{\partial xz} \quad (6)$$

$$H_0 = sin^2(\theta)\frac{\partial^2}{\partial x^2} + cos^2(\theta)\frac{\partial^2}{\partial z^2} + sin(2\theta)\frac{\partial^2}{\partial xz} \quad (7)$$

where θ is the angle of the symmetry axis with respect to the z axis. Unlike the VTI equations as shown in Zhou *et al.* (Zhou et al., 2006b), cross-derivative terms appear in equations (6) and (7) because of the TTI characteristics. In the following, we will extend the 2D TTI case into 3D TTI by axis rotations of the original VTI equations. We can derivate a 3D anisotropic acoustic wave equation for TTI media by combining two rotations. The first one is a rotation of angle θ along y-axis, and the second rotation along angle ϕ in the x-axis. We apply this variable change to the 3D anisotropic acoustic equation for VTI media (see Annexe A for more details). The 3D anisotropic acoustic equation for TTI media can be formulated as in 2D [equations (4) and (5)] but with the following expressions for the two differential operators H and H_0.

$$\begin{aligned} H &= A\frac{\partial^2}{\partial x^2} + (BE+D)\frac{\partial^2}{\partial y^2} + (BD+E)\frac{\partial^2}{\partial z^2} \\ &\quad -CH\frac{\partial^2}{\partial x\partial y} - CG\frac{\partial^2}{\partial x\partial z} - AF\frac{\partial^2}{\partial y\partial z} \quad (8) \\ H_0 &= B\frac{\partial^2}{\partial x^2} + AE\frac{\partial^2}{\partial y^2} + AD\frac{\partial^2}{\partial z^2} \\ &\quad +CH\frac{\partial^2}{\partial x\partial y} + CG\frac{\partial^2}{\partial x\partial z} + AF\frac{\partial^2}{\partial y\partial z} \quad (9) \end{aligned}$$

with

$$A = cos^2(\theta),\quad B = sin^2(\theta),\ C = sin(2\theta),\ D = cos^2(\phi),$$
$$E = sin^2(\phi),\quad F = sin(2\phi),\ G = cos(\phi),\ H = sin(\phi)$$

In equations (8) and (9), if we set θ then ϕ to zero, we can check that the formulation of the differential operators is reduced almost like for the 2D TTI case [equations (6) and (7)]. To treat the difference, there is an additional second derivative which corresponds to the third direction not tilted (y in the case $\theta = 0$ and (x in the case $\phi = 0$). Thus, in numerical experiments, we should observe results similar to 2D TTI ones for those particular values of θ and ϕ. Note that in general, θ and ϕ are function of the space but assuming they vary slowly, their derivatives can be neglected in the equations.

TIME AND SPATIAL DISCRETIZATION

In this section we describe the time and spatial discretization for the wave propagation in 3D TTI media. The isotropic case and 3D VTI case are a simplification of the following. The second-order FD stencil of equations (4) and (5) in time direction can be symbolically written as:

$$p^{n+1} = 2p^n - p^{n-1} + c^2\Delta t^2\left((1+2\delta)H(p^n+q^n) + H_0 p^n\right) \quad (10)$$

$$q^{n+1} = 2q^n - q^{n-1} + c^2\Delta t^2\left(2(\varepsilon-\delta)H(p^n+q^n)\right) \quad (11)$$

To reduce numerical dispersions, the space derivatives in equations should be approximated by higher order schemes. Obviously, because of the cross-derivative term, FD is found to be too complex for implementation. And because the 3D FFT is hard to be implemented efficiently, we decided to use hybrid-pseudospectral algorithms. i.e., we choose to apply pseudospectral algorithm in lateral directions (x and y) and FD scheme in depth direction (z). In this way, we remove the need to compute the cross derivatives.

The hybrid formulation relies on the following algorithm:
First, we compute the partial forward FT in x and y of p^n and q^n : $p^n(x,y,z,t) \to \hat{p}^n(x,y,z,t)$, $q^n(x,y,z,t) \to \hat{q}^n(x,y,z,t)$.
Second, we calculate the derivatives in x and y needed in H and in the wave number domain. We define :

$$\begin{aligned} \hat{H}_1 &= (-Ak_x^2 - (BE+D)k_y^2 + CHk_xk_y)(\hat{p}^n + \hat{q}^n), \\ \hat{H}_2 &= -(CGik_x + AFik_y)(\hat{p}^n + \hat{q}^n) \\ \hat{H}_{01} &= -(Bk_x^2 + AEk_y^2 + CHk_xk_y)\hat{p}^n \\ \hat{H}_{02} &= (CGik_x + AFik_y)\hat{p}^n \end{aligned}$$

Third, we calculate the backward FT of $\hat{H}_1$, $\hat{H}_2$, $\hat{H}_{01}$ and $\hat{H}_{02}$.
Then equations (7,8) give

$$\begin{aligned} p^{n+1} &= 2p^n - p^{n-1} + c^2\Delta t^2\Bigg((1+2\delta)\left[H_1 + \tfrac{\partial H_2}{\partial z}\right. \\ &+ \left.(BD+E)\tfrac{\partial^2(p^n+q^n)}{\partial z^2}\right] + H_{01} + \tfrac{\partial H_{02}}{\partial z} + AD\tfrac{\partial^2 p^n}{\partial z^2}\Bigg) \quad (12) \end{aligned}$$

$$\begin{aligned} q^{n+1} &= 2q^n - q^{n-1} + c^2\Delta t^2\Bigg(2(\varepsilon-\delta)\left[H_1 + \tfrac{\partial H_2}{\partial z}\right. \\ &+ \left.(BD+E)\tfrac{\partial^2(p^n+q^n)}{\partial z^2}\right]\Bigg) \quad (13) \end{aligned}$$

To finish, we calculate the derivatives in z with FD schemes.

NUMERICAL RESULTS

To validate our hybrid pseudospectral-FD method for 3D RTM using the two-way acoustic wave equation for diverse media cases, we run impulse response experiments for an homogeneous isotropic media, an homogeneous VTI media and an homogeneous TTI media. The given VTI/TTI media have the anisotropic parameters defined by c=2000m/s, $\varepsilon = 0.2$ and $\delta = 0.05$. The TTI media is considered with various rotation angles θ and ϕ of the symmetry axis. The source wavelet is Ricker wavelet with a peak of frequency as 30 Hz. Figure 1 shows the snapshots generated for the isotropic case ($\varepsilon = \delta = 0$; $\theta = \phi = 0$) and for the anisotropic case ($\varepsilon = 0.2$, $\delta = 0.05$) with the following combinations of the two angles ($\theta = \phi = 0$ (VTI case), $\theta = \pi/4; \phi = 0$, $\theta = \pi/2; \phi = 0$, $\theta = \phi = \pi/4$). Figure 2 shows the results of impulse responses of pre-stack migration for the same angle combinations as presented for the modeling. The location for the source and receiver are 500m apart. Figures 1 and 2 show 2D cuts in the middle of y dimension of the domain. We have checked that the impulse responses are the same in the mentioned 2D cut with $\theta = \alpha$, $\phi = 0$, and in the middle of x dimension with $\theta = 0$, $\phi = \alpha$. These results are consistent with the ones obtained by Zhou *et al.* in 2D TTI. The artifacts in the center of the image, for some particular cases, exist due to the slower-traveling waves inherent in the solution.

CONCLUSION

We have described a 3D reverse-time migration with Hybrid Finite Difference-pseudospectral Method. The anisotropic wave equation for 3D TTI media is an extension of 3D VTI media (Zhou et al., 2006b) and 2D TTI media equations (Zhou et al., 2006a) proposed by Zhou *et al.* This anisotropic wave equation provides a good kinematics approximation to the elastic equation. It uses lower-order derivatives than Alkhalifah's, and it is convenient not only for computationally efficiency but also for modeling and migration algorithm. The use of pseudospectral method to compute the derivatives in the lateral directions is appealing because it corresponds to an high order accuracy scheme, and at the same time allows us to use coarse meshes, also simplifies the computation of the cross derivatives in the case of 3D TTI media. Impulse response experiments for modeling and pre-stack migration show the validity of the algorithm.

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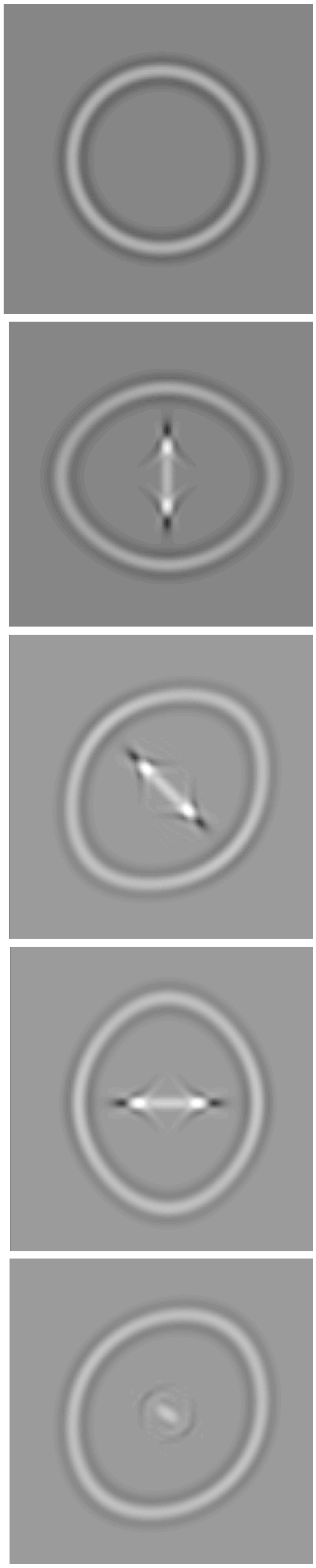

Figure 1: *Impulse responses of pre-stack migration. From top to bottom, they correspond to isotropic media, 3D VTI media, 3D TTI media with diverse angle combinations* $\theta = \pi/4; \phi = 0$, $\theta = \pi/2; \phi = 0$ and $\theta = \pi/4; \phi = \pi/4$.

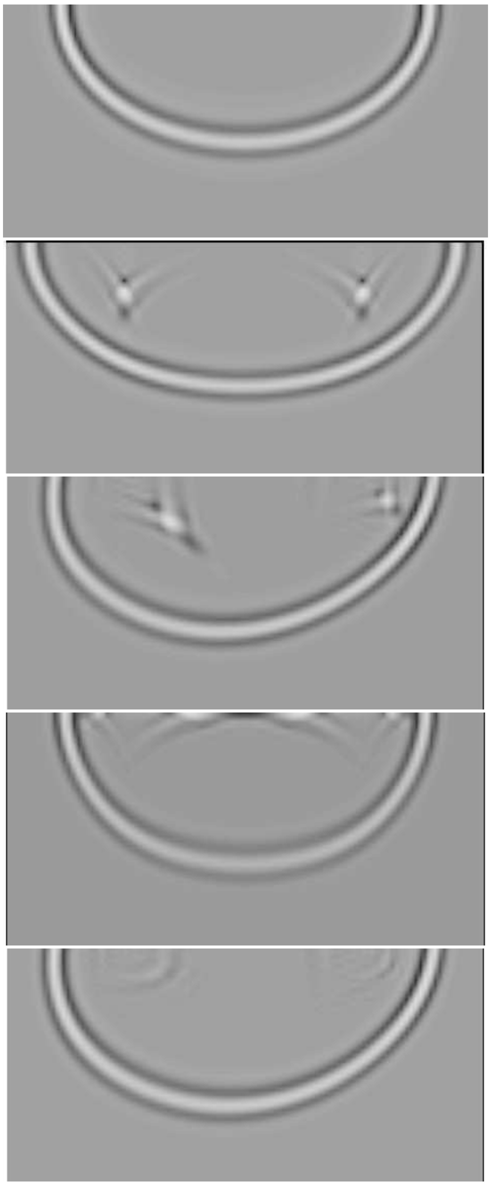

Figure 2: *Impulse responses of pre-stack migration. From top to bottom, they correspond to isotropic media, 3D VTI media, 3D TTI media with diverse angle combinations* $\theta = \pi/4; \phi = 0$, $\theta = \pi/2; \phi = 0$ and $\theta = \pi/4; \phi = \pi/4$.

ANNEXE A

We can remark that 2D equations for TTI media can be deduced from 2D equations for VTI media by a variable change corresponding to a rotation of the z axis in the counter clockwise sense. The corresponding rotation matrix A writes in 2D plane (x,z) :

$$A = \begin{pmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta) \end{pmatrix}$$

In 3D space (x,y,z), A writes :

$$A = \begin{pmatrix} cos(\theta) & 0 & sin(\theta) \\ 0 & 1 & 0 \\ -sin(\theta) & 0 & cos(\theta) \end{pmatrix}$$

Thus, we can write a $3D$ anisotropic acoustic wave equation for TTI media by combining two rotations, the first one being the rotation in the (x,z) plane corresponding to matrix A and a second one in the (y,z) plane corresponding to the following matrix B :

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos(\phi) & sin(\phi) \\ 0 & -sin(\phi) & cos(\phi) \end{pmatrix}$$

where ϕ is the angle of the second rotation.

The matrix $C = BA$ combining the two rotations writes:

$$C = \begin{pmatrix} cos(\theta) & 0 & sin(\theta) \\ -sin(\theta)sin(\phi) & cos(\phi) & cos(\theta)sin(\phi) \\ -sin(\theta)cos(\phi) & -sin(\phi) & cos(\theta)cos(\phi) \end{pmatrix}$$

The variable change $(x,y,z) \mapsto (x^{'},y^{'},z^{'})$writes :

$$\begin{aligned} \frac{\partial}{\partial x} &= cos(\theta)\frac{\partial}{\partial x'} - sin(\theta)sin(\phi)\frac{\partial}{\partial y'} - sin(\theta)cos(\phi)\frac{\partial}{\partial z'} \\ \frac{\partial}{\partial y} &= cos(\phi)\frac{\partial}{\partial y'} - sin(\phi)\frac{\partial}{\partial z'} \\ \frac{\partial}{\partial z} &= sin(\theta)\frac{\partial}{\partial x'} + cos(\theta)sin(\phi)\frac{\partial}{\partial y'} + cos(\theta)cos(\phi)\frac{\partial}{\partial z'} \end{aligned}$$

Then, appling this variable change to the 3D anisotropic acoustic equation for VTI media, we obtain expressions (6) and (7) for the two differential operators H and H_0.